International Journal of Applied Mathematics and Numerical Research

Solving the Time-Fractional DSW Equation Semi-Analytically by Combining the Shehu Transform with the Adomian Decomposition Method (ADM), the Modified Adomian Decomposition Method (MADM), and the New Iteration Method (NIM)

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Article Info

ISSN (Online): 3107-7110
Volume: 01
Issue: 06
Novermber - December 2025
Received: 24-09-2025
Accepted: 28-10-2025
Published: 22-11-2025
Page No: 01-17

Abstract

This manuscript employs the Shehu Transform in combination with the Adomian Decomposition Method (ADM), the Modified Adomian Decomposition Method (MADM), and the New Iteration Method (NIM) to obtain a series solution for the time-fractional Drinfeld-Sokolov-Wilson (DSW) equations. These equations model gravitational water flows affected by shear stress, covering situations like flows through vegetation, overland flows, dam breaks, and floods. To verify the accuracy of the solutions derived using the Shehu Transform Adomian Decomposition Method (STADM) and the Shehu Transform Modified Adomian Decomposition Method (STMADM), the relative absolute error for the series solutions is calculated for $\alpha=1$, where α represents the fractional time derivative and lies within the range $\alpha<1$. The Shehu transform Modified Adomian Decomposition Method (STMADM) generally provides a better approximation with a smaller error than the standard STADM. Finally, the results obtained by the Homotopy Perturbation Method (HPM) and the New Iteration Method (NIM) are fairly comparable to the solutions provided by the STMADM. This suggests that the outcomes generated by STMADM and the Shehu Transform New Iteration Method (STNIM) are very similar. Using both STMADM and STNIM to time-fractional differential equations yields comparable and reliable results, it can be inferred.

Keywords: Shehu Transform (ST), Adomian Decomposition Method (ADM), Caputo Operators and Fractional Partial Differential Equations (FPDE)

1. Introduction

Nonlinear partial differential equations (NPDEs), are used to describe a wide variety of complex occurrences in the sciences, especially in the physical sciences. Finding explicit and accurate solutions, particularly for solitary wave solutions to nonlinear evolution equations in mathematical physics, is a crucial part of nonlinear research.

The Drinfeld-Sokolov-Wilson (DSW) model, introduced by Drinfeld, Sokolov, and Wilson, represents a system of nonlinear partial differential equations (NLPDEs) designed to simulate gravitational water flow influenced by shear stress, such as overland flows, dam breaks, flows through vegetation, and floods (Drinfeld V. G. *et al.*, 1981; Drinfeld V. G. *et al.*, 1985) [8-9]. Wilson (Wilson, G., 1982) [22] further expanded this model to account for dispersive water waves, which play a significant role in fluid dynamics. Hirota *et al.* later presented unique static solitons for this equation that interact with moving solitons without any deformation (Hirota, R., *et al.*, 1986) [12].

$$D_t V(x,t) = -aW_x(x,t)$$

$$\mathcal{D}_t \mathcal{W}(x,t) = -b\mathcal{W}_{xxx}(x,t) - c\mathcal{V}(x,t)\mathcal{W}_x(x,t) - d\mathcal{W}(x,t)\mathcal{V}_x(x,t). \quad (1)$$

Here, t is an independent variable, while a, b, c, and d are non-negative parameters that represent the amplitude of wave modes. The functions V(x, t) describe the amplitude of wave modes as they vary with time t and space x, respectively. Every NLPDE has an infinite number of solutions, and finding the precise solution is a challenging undertaking. Now that more potent computers and sophisticated computational methods have been developed, it is possible to partially treat even nonlinear problems. Several researchers have recently shown an interest in this standard DSW model (Hirota, R., *et al*., 1986; Li, Z. B. 2007; Inc, M. 2006; Wazwaz, A. M. 2006; Zha, X. Q, *et al*., 2008; Guo, G. X., *et al*., 2010; Zhang, W. M. 2011; Ali, N., *et al*., 2022) [12, 13, 3, 11, 21, 23-24].

Many more partial differential equations (PDEs), similar to the set of PDEs mentioned above, are essential for explaining a wide range of mathematical and physical events. The presence of fractional-order derivatives in nonlinear partial differential equations (NLPDEs) makes research more interesting and difficult; as a result, these equations are referred to as fractional partial differential equations (FPDEs). Jaradat *et al*. (Jaradat *et al*., 2016) [14] generalized the Drinfeld Sokolov-Wilson (DSW) system by replacing the first-order time derivative in the equations with a fractional derivative of order α. This generalized system, referred to as the fractional DSW system and has important applications in both physics and engineering.

$$\mathcal{D}_t^{\alpha} \mathcal{V}(x,t) = -a\mathcal{W}(x,t)\mathcal{W}_x(x,t),$$

$$\mathcal{D}_t^{\alpha} \mathcal{W}(x,t) = -b\mathcal{W}_{xxx}(x,t) - c(x,t)\mathcal{W}_x(x,t) - d\mathcal{W}(x,t)\mathcal{V}_x(x,t), t > 1, 0 < \alpha \leq 1. \quad (2)$$

It has made good progress to solve nonlinear fractional/non-fractional partial differential equations with approximate analytical approaches. Numerous researchers have extensively studied the DSW equations to find both exact and approximate solutions for fractional and non-fractional orders, such as: Homotopy perturbation transform method (HPM) (Singh, P. K., *et al*., 2015) [19], Extended tanh method (Bashar, M. H., *et al*., 2023) [4] and singular operators method (Saifullah, S., *et al*., 2022) [17] etc. In addition to these approaches, there are several mathematical strategies are also available for solving linear and nonlinear differential equations, such as the Adomian decomposition method (ADM) (Adomian G., 1994; Wazwaz, A. M., 1999) [2, 20] and New iteration method (NIM) (Daftardar-Gejji, V., *et al*., 2006) [7]. Alternatively referred to as the inverse operator approach, one of the better methods for solving nonlinear differential equations is the integral transformation, which is comparable to the semi-analytical approaches that were previously mentioned. Saifullah (Saifullah *et al*., 2022) [17] solved fractional DSW equations with the Laplace transform combined with the Adomian decomposition method, commonly known as (LADM). This study aims to address the challenges posed by the unreported findings that have yet to be identified in the field. The primary focus of this research is to determine the solutions of time-fractional DSW equations using Caputo fractional derivatives. To achieve this, we employ three advanced mathematical methods: the Shehu Transform Adomian Decomposition Method (STADM) the Shehu Transform Modified Adomian Decomposition Method (STMADM), and the Shehu Transform New Iteration Method (STNIM). This research is the first to comprehensively explore the Shehu Transform Adomian Decomposition Method (STADM) in relation to Mittag-Leffler type kernels and fractal fractional operators exhibiting exponential decay. These methods offer new insights into the behavior and solutions of complex time-fractional differential equations while enhancing both accuracy and efficiency. The structure of the paper is as follows: While Sections 3 and 4 provide a detailed explanation of the stages involved in the suggested methods, Section 2 provides basic definitions for fractional calculus. We use the recommended techniques in Section 5 to get approximations for the time-fractional DSW system solutions. Lastly, our findings and observations are presented in Section 6.

2. Basics of Shehu Transformation

We provide some fundamental concepts and characteristics of the theory of fractional calculus, which are used throughout the remainder of this piece of writing.

Definition 2.1.: Consider the following set $X = \{f(t): \exists \mathcal{M}, v_1, v_2 > 0, |f(t)| < \mathcal{M} e^{\frac{|t|}{v_i}}, \text{if}, t \in (-1)^i \times [0, \infty)\}$
Next, the Shehu transform of $f(t) \in X$ may be expressed as follows:

$$\mathcal{H}[f(t)] = \mathcal{H}(u,s) = s\int_0^{\infty} f(t) e^{\frac{-st}{u}} dt. \quad (3)$$

The Inverse Shehu transform is also provided as:

$$\mathcal{H}^{-1}[F(s,u)] = f(t), \text{ for}, t \geq 0.$$

Equivalently,

$$f(t) = \mathcal{H}^{-1}[F(s,u)] = \frac{1}{2\pi\iota}\int_{\alpha - i\infty}^{\alpha + i\infty} \frac{1}{u} e^{\frac{st}{u}} F(s,u) ds.$$

In the complex plane, i.e. (s=x+iy), this integral is considered along s= α. Here, s and u are the Shehu transform variables, while α is a real constant. For more details, see (Maitama, S., *et al.*, 2019) [16].

Definition 2.2.: The Caputo time derivative regarding t of order β>0 of v(x, t) in $H^1(a, b)$ is defined as (Adams, R. A. 1975: Belgacem, R., *et al.*, 2019) [1, 6]:

$$\mathcal{D}_t^{\beta} v(x,t) = \begin{cases} \frac{1}{\Gamma(n-\beta)} \int_0^t (t-\zeta)^{n-\beta-1} \frac{\partial^n v(x,\zeta)}{\partial \zeta^n} d\zeta, \text{ if}, n-1<\beta<n, n \in N \\ \frac{\partial^n v(x,\zeta)}{\partial \zeta^n}, if, \beta = n \end{cases} \tag{4}$$

Definition 2.3.: The Shehu transform of t^m, i.e. (Maitama, S., *et al.*, 2019) [16]:

$$\mathcal{H}\left(\frac{t^m}{m!}\right) = \left(\frac{u}{s}\right)^m, m = 0, 1, 2, \ldots$$

And

$$\mathcal{H}\left(\frac{t^{\beta}}{\Gamma(\beta+1)}\right) = \left(\frac{u}{s}\right)^{\beta}, \beta \succ -1.$$

Definition 2.4: The Shehu Transform (Maitama, S. *et al.*, 2019) [16] of $\mathcal{D}_t^{\,n} v(x,t)$ regarding t of order n ≥1:

$$\mathcal{H}[\mathcal{D}_t^{\,n} v(x,t)] = \left(\frac{s}{u}\right)^{n} \bar{v}(x,u) - \sum_{k=0}^{n-1} \left(\frac{s}{u}\right)^{n-k-1} \mathcal{D}_t^{\,n} v(x,t)|_{t=0}, \tag{5}$$

where $\bar{v}(x,u)$ is Shehu Transform of v(x, t).

Definition 2.5: The Shehu Transform (Belgacem, R., *et al.*, 2019) [6] of $\mathcal{D}_t^{\beta} v(x,t)$ regarding t of order β >0:

$$\mathcal{H}\left[\mathcal{D}_t^{\beta} v(x,t)\right] = \left(\frac{s}{u}\right)^{\beta} \bar{v}(x,u) - \sum_{k=0}^{n-1} \left(\frac{s}{u}\right)^{\beta-k-1} \mathcal{D}_t^{\beta} v(x,t)|_{t=0}, \tag{6}$$

where $\bar{v}(x,u)$ is Shehu Transform of v(x, t) and $\mathcal{D}_t^{\beta}$ is the Caputo derivative of order β > 0.

3. Algorithms for the DSW system

Consider the DSW system previously described:

$$\mathcal{D}_t^{\alpha} \mathcal{V}(x,t) = -a\mathcal{W}(x,t)\mathcal{W}_x(x,t),$$

$$\mathcal{D}_t^{\alpha} \mathcal{W}(x,t) = -b\mathcal{W}_{xxx}(x,t) - c(x,t)\mathcal{W}_x(x,t) - d\mathcal{W}(x,t)\mathcal{V}_x(x,t), t > 1, 0 < \alpha \le 1.$$

Where, 0 < α ≤ 1 and the initial conditions are v(x, 0) = f(x) and w(x, 0) = g(x). Where f(x) and g(x) are two known functions of the dependent variable x. To solve the aforementioned system of equations under the given initial conditions, take the ST of both sides of the system of equations as:

$$\mathcal{H}[\mathcal{D}_t^{\alpha} v(x,t)] = -a\mathcal{H}[w(x,t)w_x(x,t)],$$

$$\mathcal{H}\left[\mathcal{D}_t^{\beta} w(x,t)\right] = -b\mathcal{H}[w_{xxx}(x,t)] - c\mathcal{H}[v(x,t)w_x(x,t)] - d\mathcal{H}[w(x,t)v_x(x,t)].$$

Or

$$\left(\frac{s}{u}\right)^{\alpha} V(x,s,u) - \left(\frac{s}{u}\right)^{\alpha-1} v(x,0) = -a\mathcal{H}[w(x,t)w_x(x,t)],$$

$$\left(\frac{s}{u}\right)^{\beta} W(x,s,u) - \left(\frac{s}{u}\right)^{\beta-1} w(x,0) = -b\mathcal{H}[w_{xxx}(x,t)] - c\mathcal{H}[v(x,t)w_x(x,t)] - d\mathcal{H}[w(x,t)v_x(x,t)].$$

Or

$$V(x,s,u) = f(x)\left(\frac{u}{s}\right) - a\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[w(x,t)w_x(x,t)],$$

$$W(x,s,u) = g(x)\left(\frac{u}{s}\right) - b\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[w_{xxx}(x,t)] - c\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[v(x,t)w_x(x,t)] - d\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[w(x,t)v_x(x,t)].$$

Applying the inverse Shehu transform to above equations we get,

$$v(x,t) = f(x) - a\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha\mathcal{H}}[w(x,t)w_x(x,t)]\right],$$

$$w(x,t) = g(x) - b\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[w_{xxx}(x,t)]\right] - c\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[v(x,t)w_x(x,t)]\right] - d\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[w(x,t)v_x(x,t)]\right].$$

Or

$$v(x,t) = f(x) - a\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}[\mathcal{N}_1]\right], \tag{7}$$

$$w(x,t) = g(x) - b\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[\mathcal{L}]\right] - c\,\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[\mathcal{N}_2]\right] - d\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta}\mathcal{H}[\mathcal{N}_3]\right]. \tag{8}$$

Here, $\mathcal{N}_1$, $\mathcal{N}_2$ and $\mathcal{N}_3$ represents nonlinear terms of differential equations, while L represents the linear part of the same. Let there be a series solution to the initial differential equations as:

$$v(x,t) = \sum_{i=0}^{\infty} v_i(x,t) \text{ and } w(x,t) = \sum_{i=0}^{\infty} w_i(x,t).$$

Decomposition of non-linear terms $\mathcal{N}_1 = w(x,t)w_x(x,t), \mathcal{N}_2 = v(x,t)w_x(x,t) and\ \mathcal{N}_3 = w(x,t)v_x(x,t)$ will be made by Adomian polynomials A_n, B_n, and C_n respectively.

$$\mathcal{N}_1 = \sum_{n=0}^{\infty} A_n, \mathcal{N}_2 = \sum_{n=0}^{\infty} B_n, \mathcal{N}_3 = \sum_{n=0}^{\infty} C_n.\text{c}$$

To solve the DSW system the few Adomian polynomials are calculated for $\mathcal{N}_1 = w(x,t)w_x(x,t)$

$A_0 = w_0 (w_0)_x$

$A_1 = d/d\lambda\ [(w_0 + \lambda w_1)(w_0 + \lambda w_1)_x]\ |_{\lambda=0}$

$= w_1(w_0)_x + w_0(w_1)_x$

$A_2 = (1 / 2!) * d^2/d\lambda^2\ [(w_0 + \lambda w_1 + \lambda^2 w_2)(w_0 + \lambda w_1 + \lambda^2 w_2)_x]\ |_{\lambda=0}$

$= w_2(w_0)_x + w_1(w_1)_x + w_0(w_2)_x$

$A_3 = (1 / 3!) * d^3/d\lambda^3\ [(w_0 + \lambda w_1 + \lambda^2 w_2 + \lambda^3 w_3)(w_0 + \lambda w_1 + \lambda^2 w_2 + \lambda^3 w_3)_x]\ |_{\lambda=0}$

… =...

In general, Adomian Polynomials for N1 are calculated as below

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} N_1\left(\left(\sum_{k=0}^{n} \lambda^k w_k\right)\left(\sum_{k=0}^{n} \lambda^k (w_k)_x\right)\right)|_{\lambda=0}. \tag{9}$$

Similarly, we have Adomian polynomials for $\mathcal{N}_2 = v(x,t)w_x(x,t)$as follows:

$B_0 = v^0(w^0)_x$

$B_1 = d/d\lambda[(v^0 + \lambda v^1)(w^0 + \lambda(w^1)_x)]|_{\lambda=0}$

$= v^1(w^0)_x + v^0(w^1)_x$

$B_2 = (1/2!) * d^2/d\lambda^2[(v^0 + \lambda v^1 + \lambda^2 v^2)(w^0 + \lambda w^1 + \lambda^2 w^2)_x]|_{\lambda=0}$

$= v^2(w^0)_x + v^1(w^1)_x + v^0(w^2)_x$

... = ...

In general, Adomian Polynomials for N2 are calculated as below:

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} N_2\big((\sum_{k=0}^{n} \lambda^k v_k)(\sum_{k=0}^{n} \lambda^k (w_k)_x)\big)|_{\lambda=0}. \tag{10}$$

Similarly, we have Adomian polynomials for $\mathcal{N}_3 = \mathrm{w(x,t)v_x(x,t)}$as follows:

$C_0 = w_0\,(v_0)_x$

$C_1 = d/d\lambda\,[\,(w_0 + \lambda w_1)(v_0 + \lambda v_1)_x\,]|_{\lambda=0}$

$= w_1(v_0)_x + w_0(v_1)_x$

$C_2 = (1\,/\,2!) * d^2/d\lambda^2\,[\,(w_0 + \lambda w_1 + \lambda^2 w_2)(v_0 + \lambda v_1 + \lambda^2 v_2)_x\,]\,|_{\lambda=0}$

$= w_0(v_2)_x + w_1(v_1)_x + w_2(v_0)_x$

... =...

In general, Adomian Polynomials for N3 are calculated as below:

$$C_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} N_3\big((\sum_{k=0}^{n} \lambda^k w_k)(\sum_{k=0}^{n} \lambda^k (v_k)_x)\big)|_{\lambda=0}. \tag{11}$$

To obtain a series solution following recurrence formula will be used,

$$v_n(x,t) = f(x) - a\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[A_{n-1}]\right],$$

$$w_n(x,t) = g(x) - \; b\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta} \mathcal{H}[\mathcal{L}]\right] - \; d\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta} \mathcal{H}[C_{n-1}]\right] \tag{12}$$

4. Implementation of STADM

By employing the suggested methodologies as described below, we can derive the series solutions for the fractionally coupled DSW problem.

$$\mathcal{D}_t^{\alpha}\mathcal{V}(x,t) = -3\mathcal{W}(x,t)\mathcal{W}_x(x,t),$$

$$\mathcal{D}_t^{\alpha}\mathcal{W}(x,t) = -2\mathcal{W}_{xxx}(x,t) - 2(x,t)\mathcal{W}_x(x,t) - \mathcal{W}(x,t)\mathcal{V}_x(x,t), t \; > \; 1, 0 \; < \; \alpha \leq 1. \tag{13}$$

subjected to the initial guesses conditions:

$$\mathcal{V}(x,0) = 3\sec h^2 x \; and \; \mathcal{W}(x,0) = 2\sec h\, x.$$

For α = 1, this system of equations has exact solution as:

$$\mathcal{V}(x,t) = 3sech^2(x-2t),$$

$$\mathcal{W}(x,t) = 2sech(x-2t).$$

As explained in the section 3 the iteration scheme for calculating the series components is as follows:

$$v_i(x,t) = 3\sec h^2 x - 3\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[\textstyle\sum_{i=0}^{\infty} A_i]\right],$$

$$w_i(x,t) = 2\sec h\, x \; - \; b\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta} \mathcal{H}[\mathcal{L}_i]\right] - \; c\,\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta} \mathcal{H}[\textstyle\sum_{i=0}^{\infty} B_i]\right] - d\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta} \mathcal{H}[\textstyle\sum_{i=0}^{\infty} C_i]\right].$$

Let, $\mathrm{v_0(x,t) = 3\sec h^2 x}$ and $\mathrm{w_0(x,t) = 2\sec h\,x}$.
The few terms are calculated below:

$$\mathcal{V}_1(x,t) = -3\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[-4\mathrm{sech}^2(x)\tan h\, x\right]\right]$$

$$= 12\mathrm{sech}^2(x)\tan h\, x\frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

$$W_1(x,t) \;=\; 2\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[2\tan h\, x\,(\tan h^2 x - 5\mathrm{sech}^4(x)\right]\right] +$$

$$2\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[6\tan h\, x\,\mathrm{sech}^3(x)\right]\right]2\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[12\tan h\, x\,\mathrm{sech}^3(x)\right]\right]$$

$$= 4\tanh(x)\sec h\,(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

Similarly,

$$V_2(x,t) = -3\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[8\left(2\sec h^2\,(x) - 3\sec h^4\,(x)\right)\right]\right]$$

$$= 24\left(-3\sec h^4\,(x) + 2\sec h^2\,(x)\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}.$$

$$\mathcal{W}_2(x,t) = 2\mathcal{H}^{-1}\left(\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)}\left(4\left(-\tanh^4(x)\,sech(x) + 18\tanh^2(x)\,sech^3 - 5sech^5(x)\right)\right)\right]\right)$$

$$+2\mathcal{H}^{-1}\left(\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)}\cdot 6\tanh(x)\,sech^3(x)\right]\right)$$

$$+\mathcal{H}^{-1}\left(\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}\left[-12\,sech^3(x)\cdot\frac{t^{\alpha}}{\Gamma(\alpha+1)}, -24sech^3(x)\left(3 - 4sech^3(x)\right)\right]\right)$$

$$= 8\left(-2\,sech^3(x) + sech(x)\right)\cdot\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

The same procedure can be used to calculate other series terms. The whole solution is now written out as the sum of these terms:

$$v(x,t) = 3sech^2(x)\left[1 + 4\,\mathrm{ta\,n}\,h\,x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 8(-3\sec h^2\,(x) + 2)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots\right] \tag{14}$$

$$w(x,t) = 2\mathrm{se\,c}\,h\,(x)\left[1 + 2\mathrm{ta\,n}\,h\,x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 4(-2\sec h^2\,(x) + 1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots\right]. \tag{15}$$

The series of solutions described above had previously been determined by many researchers using several approaches see (Baskonus, H. M. 2019) [5].

Theorem 1: The solution $w(x,t) = \sum_{i=0}^{\infty} w_i(x,t)$ using ADM is convergent if 0<k<1 and $\|w_n\| < \infty$ where k = $\frac{(L_1+L_2)t^{\alpha}}{\Gamma(\alpha+1)}$ where L_1 is the Lipschitz constant for a linear term and L_2 is lipschtiz constant for the nonlinear term. For more details see (Baskonus, H. M. 2019) [5].

4. Solution with Modified Adomian Polynomials

Since the nonlinear linear term will be broken down using the modified Adomian polynomial, this technique will be known as the Shehu Transform Modified Adomian Decomposition Method (STMADM). As this is explained in (Wazwaz, A. M. (1999) [20] the modified Adomian polynomials are calculated as below**:**

$$\overline{\mathrm{A_n}} = \mathrm{f}(\mathrm{S_n}) - \sum_{\mathrm{i}=0}^{\mathrm{n}-1}\mathrm{A_i} \tag{16}$$

where $\mathrm{S_n} = \sum_{\mathrm{i}=0}^{\mathrm{n}}\mathrm{w_i}{}_{\sum_{\mathrm{i}=0}^{\mathrm{n}}(\mathrm{w_i})_{\mathrm{x}}}$ and f is a symbol for the nonlinear part.
The few revised polynomials for N1 is $\overline{A_0} = w_0(w_0)_x$

$$\overline{A_1} = \mathcal{N}_1\left((w_0 + w_1)(w_0 + w_1)_x\right) - \overline{A_0}$$

$$= \left((w_0 + w_1)(w_0 + w_1)_x\right) - w_0(w_0)_x$$

$$= w_1(w_0)_x + w_0(w_1)_x + w_1(w_1)_x.$$

$$\overline{A_2} = \mathcal{N}_1\big((w_0 + w_1 + w_2)(w_0 + w_1 + w_2)_x\big) - (\overline{A_0} + \overline{A_1})$$

$$= \big((w_0 + w_1 + w_2)(w_0 + w_1 + w_2)_x\big) - (w_0(w_0)_x + w_1(w_0)_x + w_0(w_1)_x + w_1(w_1)_x)$$

$$= w_0(w_2)_x + w_1(w_2)_x + w_2(w_1)_x + w_2(w_0)_x + w_2(w_2)_x$$

Similarly, this is calculated for N_2

$$\overline{B_0} = v_0(w_0)_x$$

$$\overline{B_1} = \mathcal{N}_1\big((v_0 + v_1)(w_0 + w_1)_x\big) - \overline{B_0}$$

$$= \big((v_0 + v_1)(w_0 + w_1)_x\big) - v_0(w_0)_x$$

$$= v_1(w_0)_x + v_0(w_1)_x + v_1(w_1)_x$$

$$\overline{B_2} = \mathcal{N}_1\big((v_0 + v_1 + v_2)(w_0 + w_1 + w_2)_x\big) - (\overline{B_0} + \overline{B_1})$$

$$= \big((v_0 + v_1 + v_2)(w_0 + w_1 + w_2)_x\big) - (v_1(w_0)_x + v_0(w_1)_x + v_1(w_1)_x + v_0(w_0)_x)$$

$$= v_0(w_2)_x + v_1(w_2)_x + v_2(w_2)_x + v_2(w_1)_x + v_2(w_0)_x$$

Similarly, this is calculated for N3

$$\overline{C_0} = w_0(v_0)_x$$

$$\overline{C_1} = \mathcal{N}_1\big((w_0 + w_1)(v_0 + v_1)_x\big) - \overline{C_0}$$

$$= \big((w_0 + w_1)(v_0 + v_1)_x\big) - w_0(v_0)_x$$

$$= w_0(v_1)_x + w_1(v_0)_x + w_1(v_1)_x$$

$$\overline{C_2} = \mathcal{N}_1\big((w_0 + w_1 + w_2)(v_0 + v_1 + v_2)_x\big) - (\overline{C_0} + \overline{C_1})$$

$$= \big((w_0 + w_1 + w_2)(v_0 + v_1 + v_2)_x\big) - (w_0(v_1)_x + w_1(v_0)_x + w_1(v_1)_x + w_0(v_0)_x)$$

$$= w_0(v_2)_x + w_1(v_2)_x + w_2(v_1)_x + w_2(v_0)_x + w_2(v_2)_x$$

Then the revised iteration scheme is as follows

$$v_n(x,t) = f(x) - a\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[\overline{A_{n-1}}]\right], \tag{17}$$

$$w_n(x,t) = g(x) \; - \; b\,\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\beta} \mathcal{H}[L]\right] \; - \; c\,\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[\overline{B_{n-1}}]\right] \; - \; d\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[\overline{C_{n-1}}]\right] \tag{18}$$

Terms are calculated with modified ADM as follows:
Let,

$v_0(x,t) = 3\sec h^2 x$ and $w_0(x,t) = 2\sec h\, x$.

$$\mathcal{V}_1(x,t) = -3\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha} \mathcal{H}[-4\sec^2 h\, x \tan h\, x]\right]$$

$$= 12\sec^2 h\, x \tan h\, x \,\frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

$$W_1(x,t) = 2\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}[2\,tanh\,(x)\,(\,\tanh^2(x) - 5\ \ \sec^2 h\,x]\right]$$
$$+2\,\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}[6tan\,h(x)\sec^3 h\,x]\right] \quad +\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}[12\,tan\,h(x)\sec^3 h\,x]\right]$$

$$= 4\tanh(x)\ sech(x)\cdot\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

Again,

$$\mathcal{V}_2(x,t) = -3\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}[\overline{A_1}]\right],$$

$$= -3\mathcal{H}^{-1}\left[\left(\frac{u}{s}\right)^{\alpha}\mathcal{H}[w_1(w_0)_x + w_0(w_1)_x + w_1(w_1)_x]\right],$$

$$\mathcal{V}_2(x,t) = 24\big(-3\sec h^4\,(x) + 2\sec h^2\,(x)\big)$$

$$\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 48(4\sec h^3\,(x)\tanh^3(x) - 3\sec h^5\,(x)\tanh(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}.$$

$$\mathcal{W}_2(x,t) = -2(72\sec h^3\,(x)\tanh^2(x) - 5sech^5(x) - 4\sec h\,(x)\tanh^4(x))\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 2(-24\sec h^3\,(x)\tanh^2(x) + 12\sec h^5\,(x) - 12\sec h^3\,(x)\tanh^2(x))\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$-2*48(\sec h^5\,(x)\tanh(x) - \sec h^3\,(x)\tanh^3(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}$$

$$-2*12(\sec h^5\,(x) - 3\sec h^3\,(x)\tanh^3(x) +)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 2*24(\sec h^5\,(x)\tanh(x) - 2\sec h^3\,(x)\tanh^3\quad)\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}$$

$$\mathcal{W}_2(x,t) = 8\big(-2\sec h^3\,(x) + \sec h\,(x)\big)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 48(4\sec h^3\,(x)\tanh^3(x) - 3\sec h^5\,(x)\tanh(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}$$

and so on....
The three terms solution to the above problem is given below:

$$\mathcal{V}(x,t) = 3\sec h^2\,x + 12\sec^2 h\,x\tan h\,x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \big(48\sec h^2\,(x)\tanh^2(x) - 24\sec h^4\,(x)\big)$$

$$\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 48\sec h^2\,(x)\tanh(x)\,(\sec h^2\,(x) - \tanh^2(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}$$

$$\mathcal{V}(x,t) = 3\sec h^2\left[1 + 4\tan h\,x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 8(-3\sec h^2\,(x) + 2)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 48/3\tan h\,x(2\sec h^2\,(x) - 1)\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}\right] \tag{19}$$

$$\mathcal{W}(x,t) = 2\sec h\,x + 4\tanh(x)\sec h\,(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 8\big(-2\sec h^3\,(x) + \sec h\,(x)\big)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 48(4\sec h^3\,(x)\tanh^3(x) - 3\sec h^5\,(x)\tanh(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}$$

$$= \Big[2\sec\ h(x)[\,1\ +\ 2\tanh\,(x)\ \frac{t^{\alpha}}{\Gamma(\alpha+1)}\ + 4(\,-2 - 2\sec h^2\,(x) + 1)\ \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\ + 24\tan\ h\,x(4\sec h^2\,(x)\tanh^2(x)\,3\ \sec h^4\,(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}\Big] \tag{20}$$

5. Solution with NIM

Instead of decomposing the nonlinear terms with ADM and MADM, we use the iteration method as follows and decompose the term N(x, t) by New iteration approach as given below:

$$\mathcal{N}(x,t) = \sum_{n=0}^{\infty} G_n,$$

$$G_0 = \mathcal{N}(v_0)\ and$$

$$\mathcal{N}v(x,t) = \sum_{n=0}^{\infty} G_n = \sum_{n=0}^{\infty}\Big(\mathcal{N}(\sum_{i=0}^{n} v_i) - \mathcal{N}(\sum_{i=0}^{n-1} v_i)\Big), n = 1, 2, \ldots \quad (21)$$

The decomposition of nonlinear polynomials N1 is taken as follows:

$$G_0 = w_0(w_0)_x,$$

$$G_1 = \mathcal{N}_1\big((w_0 + w_1)(w_0 + w_1)_x\big) - \mathcal{N}_1(w_0(w_0)_x)$$

$$= \big((w_0 + w_1)(w_0 + w_1)_x\big) - w_0(w_0)_x$$

$$= w_1(w_0)_x + w_0(w_1)_x + w_1(w_1)_x$$

$$G_2 = \mathcal{N}_1\big((w_0 + w_1 + w_2)(w_0 + w_1 + w_2)_x\big) - \mathcal{N}_1\big((w_0 + w_1)(w_0 + w_1)_x\big)$$

$$= \big((w_0 + w_1 + w_2)(w_0 + w_1 + w_2)_x\big) - (w_0(w_0)_x + w_1(w_0)_x + w_0(w_1)_x + w_1(w_1)_x)$$

$$= w_0(w_2)_x + w_1(w_2)_x + w_2(w_1)_x + w_2(w_0)_x + w_2(w_2)_x$$

Similarly, this is calculated for N2

$$P_0 = v_0(w_0)_x$$

$$P_1 = \mathcal{N}_1\big((v_0 + v_1)(w_0 + w_1)_x\big) - \mathcal{N}_0(v_0(w_0)_x)$$

$$= \big((v_0 + v_1)(w_0 + w_1)_x\big) - v_0(w_0)_x$$

$$= v_1(w_0)_x + v_0(w_1)_x + v_1(w_1)_x$$

$$P_2 = \mathcal{N}_2\big((v_0 + v_1 + v_2)(w_0 + w_1 + w_2)_x\big) - \mathcal{N}_2\big((v_0 + v_1)(w_0 + w_1)_x\big)$$

$$= \big((v_0 + v_1 + v_2)(w_0 + w_1 + w_2)_x\big) - (v_1(w_0)_x + v_0(w_1)_x + v_1(w_1)_x + v_0(w_0)_x)$$

$$= v_0(w_2)_x + v_1(w_2)_x + v_2(w_2)_x + v_2(w_1)_x + v_2(w_0)_x$$

Similarly, this is calculated for N_3

$$K_0 = w_0(v_0)_x,$$

$$K_1 = \mathcal{N}_3\big((w_0 + w_1)(v_0 + v_1)_x\big) - \mathcal{N}_3(w_0(v_0)_x)$$

$$= (w_0 + w_1)(v_0 + v_1)_x - w_0(v_0)_x$$

$$= w_0(v_1)_x + w_1(v_0)_x + w_1(v_1)_x,$$

$$K_2 = \mathcal{N}_3\big((w_0 + w_1 + w_2)(v_0 + v_1 + v_2)_x\big) - \mathcal{N}_3\big((w_0 + w_1)(v_0 + v_1)_x\big)$$

$$= (w_0 + w_1 + w_2)(v_0 + v_1 + v_2)_x - [w_0(v_0)_x + w_0(v_1)_x + w_1(v_0)_x + w_1(v_1)_x]$$

$$= w_0(v_2)_x + w_1(v_2)_x + w_2(v_0)_x + w_2(v_1)_x + w_2(v_2)_x$$

The series components are computed using the new iteration approach as shown below:

$$\mathcal{V}_1(x,t) = 12\sec^2 h\, x \tan h\, x \frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

$$\mathcal{V}_2(x,t) = \left(48\sec h^2(x)\tanh^2(x) - 24\sec h^4(x)\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 48\sec h^2(x)\tanh(x)\left(\sec h^2(x) - \tanh^2(x)\right)\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}.$$

$$\mathcal{W}_1(x,t) = 4\tanh(x)\sec h(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

$$\mathcal{W}_2(x,t) = 8\left(-2\sec h^3(x) + \sec h(x)\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 48\left(4\sec h^3(x)\tanh^3(x) - 3\sec h^5(x)\tanh(x)\right)\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}.$$

Consequently, NIM's approximate series solution up to three terms is

$$\mathcal{V}(x,t) = 3\sec h^2\left[1 + 4\tan h\, x\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 8(-3\sec h^2(x) + 2)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 48/3\tan h\, x(2\sec h^2(x) - 1)\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}\right] \quad (22)$$

$$\mathcal{W}(x,t) = \left[2\, sec\ h(x)[\,1 + 2\, tanh\,(x)\,\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 4(-2 - 2\sec h^2(x) + 1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} 24\, tan\ h\, x(4\sec h^2(x)\tanh^2(x) - 3\sec h^4(x))\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}t^{3\alpha}\right] \quad (23)$$

This series solution matches the series solution obtained in (Ali, N., *et al*., 2022).

Theorem 2: If the function $\mathcal{N}$ is infinitely differentiable (i.e., C^{∞}) in the neighborhood of the point $\mathcal{W}_0$, and the norm $\|\mathcal{N}^n(\mathcal{W}_0)\|$ is bounded by K>0 for all n, and if the sequence $\{\mathcal{W}_i\}$ satisfies $\|\mathcal{W}_i\| \le Z < 1/e$ for i = 1, 2,..., then the series $\sum G_n$ is convergent. Additionally, for n = 1, 2..., every term in the series meets the inequality: $\|G_n\| \le K Z^n e^{n-1}(e-1)$, for n = 1, 2..., (For more details see (Ali, N., *et al*., 2022) [3].

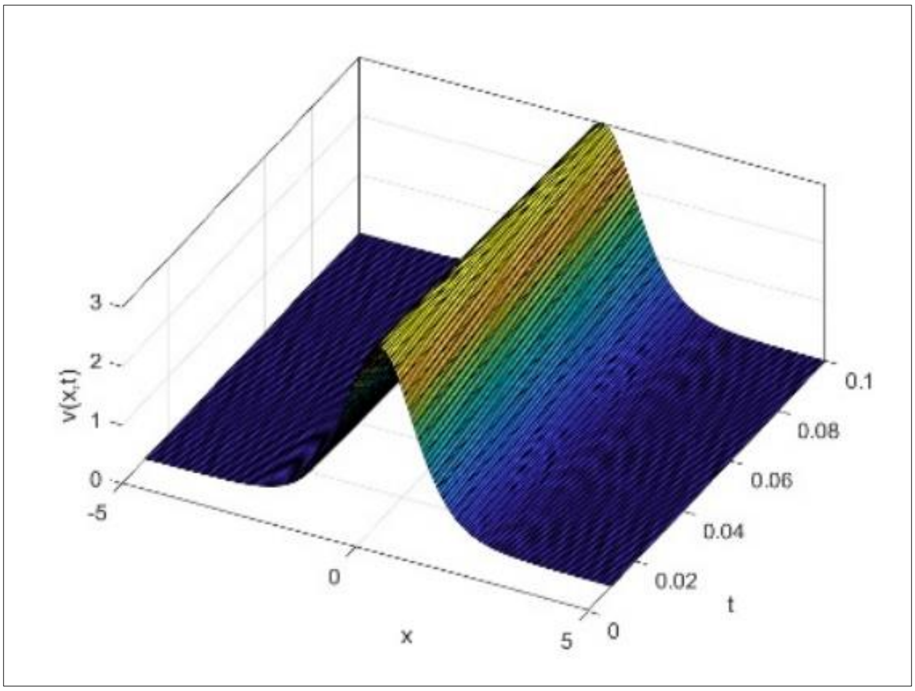

Fig 1: 3D plot of exact solutions to v for α = 1

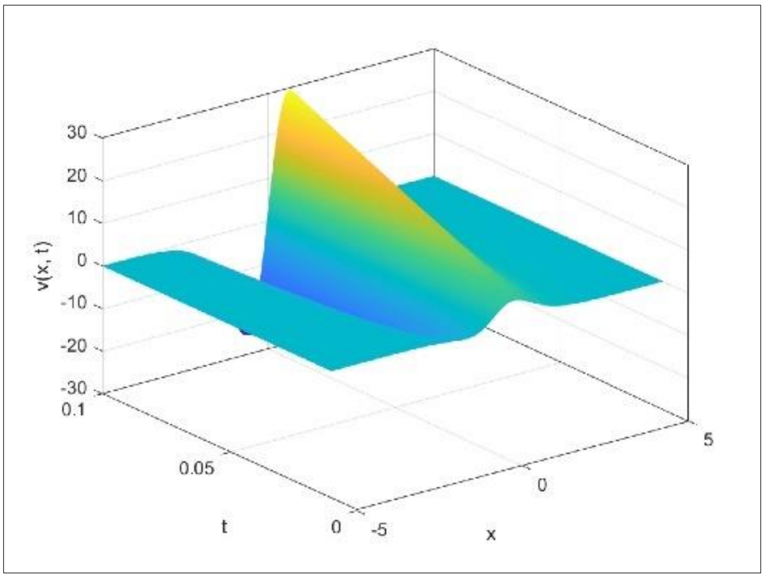

Fig 2: 3D plot of approximate solution with ADM to v for α = 1.

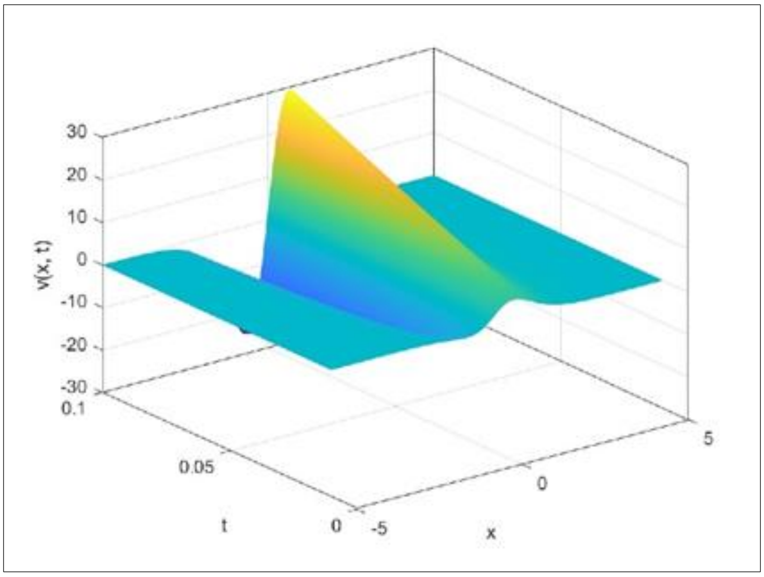

Fig 3: 3D plot of approximate solution with MADM to v for α = 1

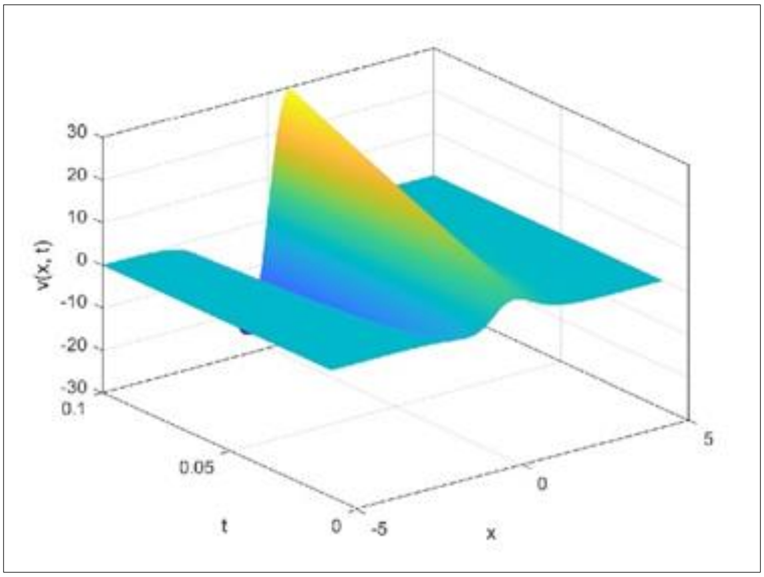

Fig 4: 3D plot of approximate solution with NIM to v for α = 1

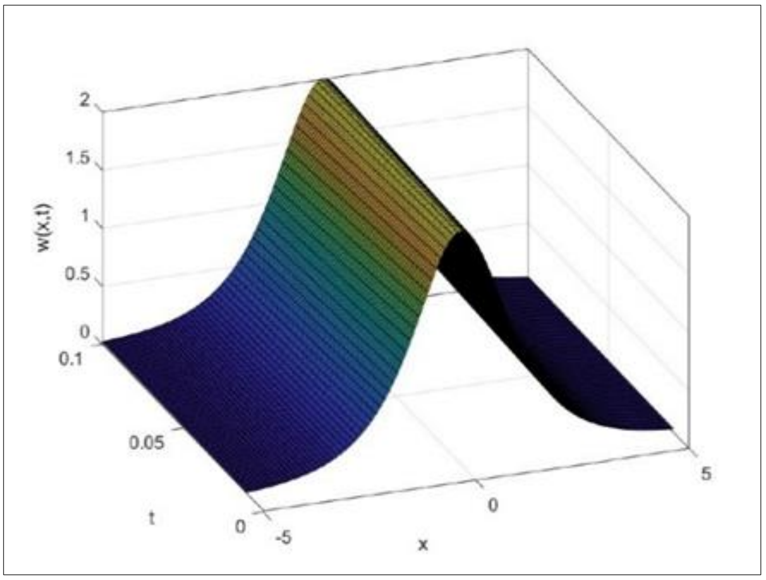

Fig 5: 3D plot of exact solution to w for α = 1

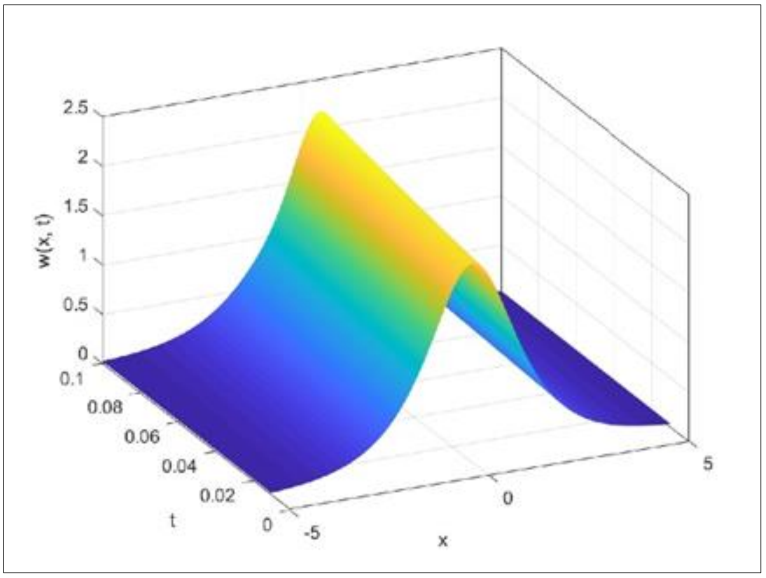

Fig 6: 3D plot of approx solution with ADM to w for α = 1.

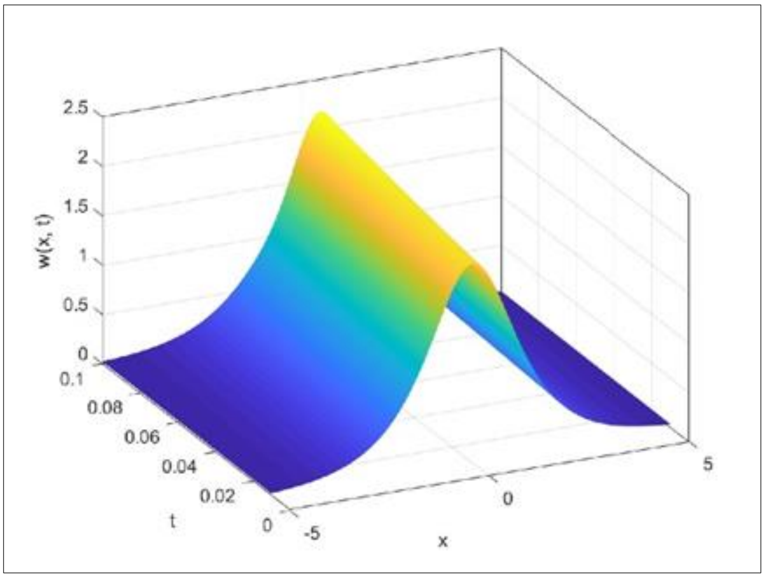

Fig 7: 3D plot of approximate solution with MADM

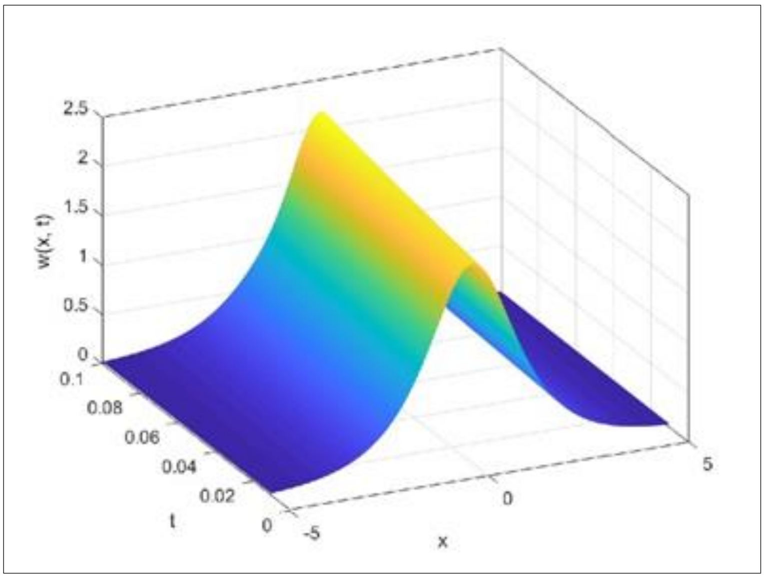

Fig 8: 3D plot of approximate solution with NIM to w for $\alpha = 1$

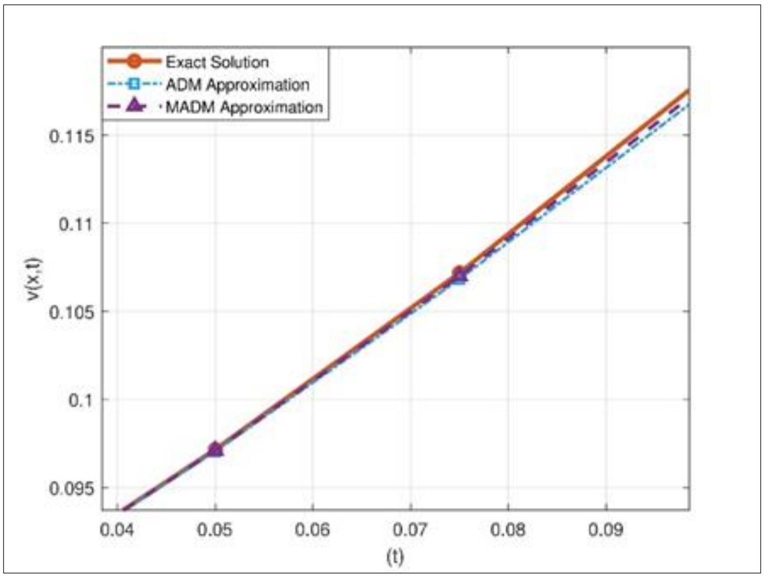

Fig 9: Proximity of series solution obtained by ADM and MADM toward the exact solution for v.

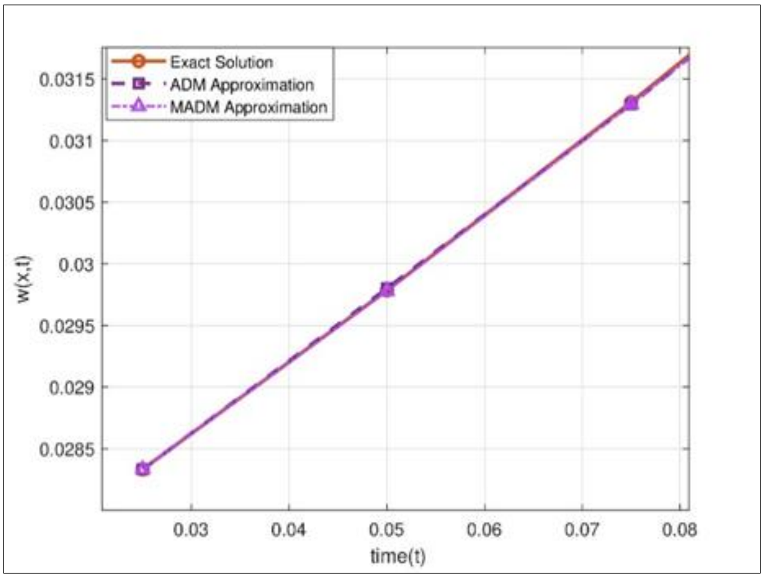

Fig 10: Proximity of series solution obtained by ADM and MADM toward the exact solution for w.

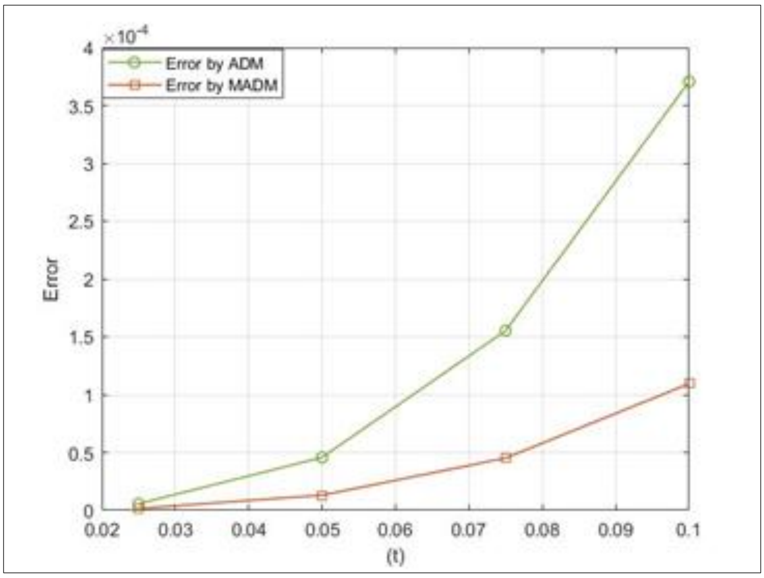

Fig 11: Propagation of absolute error for w(x, t) in case of α=1

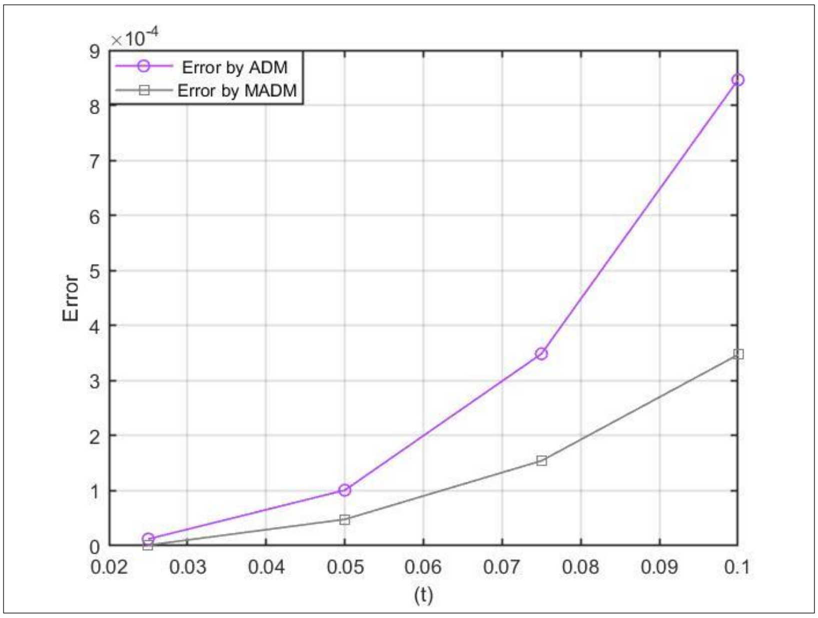

Fig 12: Propagation of absolute error for v(x, t) in case of α=1.

6. Results and Discussion

We employ several advanced mathematical techniques to develop a series solution for the time fractional Drinfeld-Sokolov-Wilson (DSW) equations. The Shehu Transform serves as a foundational tool, facilitating the application of the Adomian Decomposition Method (ADM), which is often referred to as the STADM (Singh A., *et al.*, 2023) [18]. This approach allows us to break down complex nonlinear equations into simpler components for analysis and solution. In addition, we enhance our methodology by incorporating the Modified ADM, also known as the STMADM. This modification improves the convergence rate of the series solutions, enabling a quicker approach to the true solutions of the equations at hand. Furthermore, we utilize the New Iteration Method (NIM), or the STNIM, to introduce an iterative framework that refines our solutions. By integrating these methods, we aim to achieve a robust series solution capable of accurately capturing the intricate dynamics represented by the time fractional DSW equations.

To validate the present results using three different methods (i.e., the Shehu Transform Adomian Decomposition Method (STADM), the Shehu Transform Modified Adomian Decomposition Method (STMADM), and the Shehu Transform New Iteration Method (STNIM)), the absolute errors are shown for V(x, t) and W(x, t) in Table 1. and Table 2, respectively. In these tables, the fractional order α is fixed at 1, and different values of x are considered specifically x = 2.5, 5, 7.5, 10. Additionally, various time values are examined, namely t = 0.025, 0.05, 0.075, 0.1 for both V(x, t) and W(x, t), when the solution is reduced to three terms. The examination of both tables clearly shows that the absolute error when comparing the exact solution to the approximation derived from the STMADM is notably smaller than the absolute error observed between the exact solution and the approximation obtained using the standard STADM for all specified values of t and x. Although there isn't an exact solution for fractional order α = 0.9, 0.7, 0.5, the solution using ADM and MADM has been found in Tables 3 and 4 for V(x, t) and W(x, t).

Figure 1 displays the 3D plot of the exact solution for v(x, t) in the situation of α=1, while Figures 2, 3, and 4 display the approximate solutions for the same for ADM, MADM, and NIM, respectively. Similarly, Figure 5 displays the 3D plot of the exact solution for w(x, t) in the case of α=1, while Figures 6, 7, and 8 display the approximate solutions for the same for ADM, MADM, and NIM, respectively. These figures show a comparison between the exact answer and the series solution obtained using various methods. The approximate answer we obtained using ADM, MADM, and NIM is quite similar to the actual solution, as can be inferred from both figures. The series solution obtained using STADM and STMADM is compared with the precise solution in Figures 9 and 10 for V and W, respectively. These figures show that in comparison to the solution produced by STADM, the solution obtained by STMADM is closer to the exact solution.

Figures 11 and 12 illustrate the propagation of absolute error for w and v, respectively, when x = 2.5 and α = 1. It has been determined that when solutions are produced using STMADM rather than STADM, the absolute error falls more quickly. Thus, STMADM generally yields a smaller error in series solutions than the standard STADM due to its improved handling of nonlinear terms. STMADM employs a modified approach to decompose the nonlinear part of the differential equation, which allows for better convergence and accuracy in the series expansion. By systematically incorporating corrections and refining the approximation at each step, STMADM minimizes the error more effectively than STADM, resulting in a more precise solution to the original problem. This enhanced accuracy is particularly beneficial in cases where nonlinearities significantly influence the behavior of the solution.

Iterative techniques like STADM and STMADM break down nonlinear terms into smaller, more manageable polynomial components, which makes it easier to converge to a solution. By comparing the series of solutions produced by these techniques, their efficacy is evaluated. Moreover, it has been observed that the solutions obtained through the Shehu Transform Modified Adomian Decomposition Method (STMADM) closely resemble those derived using the Homotopy Perturbation Method (HPM) as reported in the study (Baskonus, H. M. 2019), as well as those obtained through the New Iteration Method (NIM) as outlined in (Ali, N., *et al*., 2022). This similarity suggests that the STMADM and the Shehu Transform New Iteration Method (STNIM) yield comparable results in solving time-fractional differential equations. Therefore, it can be inferred that both STMADM and STNIM are consistent in their approach and effectiveness when applied to this class of equations, providing reliable solutions that align well with other established methods. The numerical values of and for various fractional orders α = 0.9, 0.7, 0.5 can be found in (Ali, N., *et al*., 2022) [3] since STADM and STNIM yield identical results.

Table 1: Absolute error analysis for v(x, t) in case of α = 1

x	t	V(exact)	V(approx. STADM)	V(approx. STMADM)	Error (STADM)	Error (STMADM)	Error (Ali, N., *et al*., 2022))
2.5	0.025	0.088042878	0.088030534	0.088036744	1.23×10^{-5}	6.13×10^{-6}	6.13×10^{-6}
	0.05	0.097151323	0.097050333	0.09710015	1.01×10^{-4}	5.13×10^{-5}	5.12×10^{-5}
	0.075	0.10718471	0.10683608	0.10700375	3.49×10^{-4}	1.81×10^{-4}	1.81×10^{-4}
	0.1	0.11823316	0.11738777	0.11778522	8.45×10^{-4}	4.48×10^{-4}	4.48×10^{-4}
5	0.025	0.00060203577	0.00060194272	0.0006019881	9.31×10^{-8}	4.77×10^{-8}	4.77×10^{-8}
	0.05	0.0006653454	0.00066458177	0.00066494477	7.64×10^{-7}	4.01×10^{-7}	4.01×10^{-7}
	0.075	0.00073531181	0.00073266682	0.00073389195	2.64×10^{-6}	1.42×10^{-6}	1.42×10^{-6}
	0.1	0.00081263476	0.00080619789	0.0008091019	6.44×10^{-6}	3.53×10^{-6}	3.53×10^{-6}
7.5	0.025	$4.05688944 \times 10^{-6}$	4.056262×10^{-6}	4.0565679×10^{-6}	6.27×10^{-10}	3.21×10^{-10}	3.21×10^{-10}
	0.05	$4.48355591 \times 10^{-6}$	4.4784066×10^{-6}	4.4808539×10^{-6}	5.15×10^{-9}	2.70×10^{-9}	2.70×10^{-9}
	0.075	$4.95509521 \times 10^{-6}$	4.9372595×10^{-6}	4.9455188×10^{-6}	1.78×10^{-8}	9.58×10^{-9}	9.58×10^{-9}
	0.1	$5.47622664 \times 10^{-6}$	5.4328205×10^{-6}	5.4523981×10^{-6}	4.34×10^{-8}	2.38×10^{-8}	2.38×10^{-8}
10	0.025	$2.73351244 \times 10^{-8}$	2.7330897×10^{-8}	2.7332958×10^{-8}	4.23×10^{-12}	2.17×10^{-12}	2.17×10^{-12}
	0.05	$3.02099845 \times 10^{-8}$	3.0175289×10^{-8}	3.0191778×10^{-8}	3.47×10^{-10}	1.82×10^{-10}	1.82×10^{-10}
	0.075	$3.33891963 \times 10^{-8}$	3.3267019×10^{-8}	3.332267×10^{-8}	1.22×10^{-9}	6.65×10^{-10}	6.65×10^{-10}
	0.1	$3.68983383 \times 10^{-8}$	3.6606088×10^{-8}	3.6738002×10^{-8}	2.92×10^{-9}	1.60×10^{-9}	1.60×10^{-9}

Table 2: Absolute error analysis for w(x, t) in case of α=1

x	t	w(exact)	W(approx. STADM)	W(approx. STMADM)	Error (STADM)	Error (STMADM)	Error (Ali, N., *et al*., 2022) [3]
2.5	0.025	0.34262298	0.3426173	0.34262138	5.68×10^{-6}	1.60×10^{-6}	1.60×10^{-6}
	0.05	0.35990985	0.35986413	0.35989677	4.57×10^{-5}	1.31×10^{-5}	1.31×10^{-5}
	0.075	0.37803829	0.37788295	0.37799312	1.55×10^{-4}	4.52×10^{-5}	4.52×10^{-5}
	0.1	0.39704435	0.39667376	0.39693491	3.71×10^{-4}	1.09×10^{-4}	1.09×10^{-4}
5	0.025	0.028332214	0.028331646	0.028331649	5.68×10^{-7}	5.65×10^{-7}	5.65×10^{-7}
	0.05	0.029784681	0.0298008	0.0297801	1.61×10^{-5}	4.58×10^{-6}	4.58×10^{-6}
	0.075	0.031311591	0.031295866	0.031295932	1.57×10^{-5}	1.57×10^{-5}	1.57×10^{-5}
	0.1	0.032916759	0.032879003	0.03287916	3.78×10^{-5}	3.76×10^{-5}	3.76×10^{-5}
7.5	0.025	0.002325765662	0.002325718991	0.002325718992	4.6671×10^{-8}	4.6670×10^{-8}	4.6670×10^{-8}
	0.05	0.002445010131	0.0024446320058	0.002444632017	3.7813×10^{-7}	3.7811×10^{-7}	3.7811×10^{-7}
	0.075	0.00257036838	0.00256907585	0.002569075886	1.29253×10^{-6}	1.29249×10^{-6}	1.29249×10^{-6}
	0.1	0.002702153868	0.002699050522	0.002699050608	3.10335×10^{-6}	3.10326×10^{-6}	3.10326×10^{-6}
10	0.025	0.0001909105353	0.0001909067042	0.0001909067042	3.8311×10^{-9}	3.8311×10^{-9}	3.8311×10^{-9}
	0.05	0.0002006987277	0.000200667689	0.0002006676891	3.10387×10^{-8}	3.10387×10^{-8}	3.10387×10^{-8}

	0.075	0.0002109887715	0.0002108826732	0.0002108826732	1.06098×10^{-7}	1.06098×10^{-7}	1.06098×10^{-7}
	0.1	0.000221806397	0.00022155165657	0.00022155165662	2.54740×10^{-7}	2.54740×10^{-7}	2.54740×10^{-7}

Table 3: Comparison of approximate values of v(x, t) at different fractional order

x	t	ADM α=0.9	MADM α=0.9	ADM α=0.7	MADM α=0.7	ADM α=0.5	MADM α=0.5
2.5	0.025	0.092566907	0.092591392	0.11161325	0.11196605	0.16658501	0.17109933
2.5	0.05	0.10518823	0.10534733	0.13721684	0.13872933	0.2204894	0.23325784
2.5	0.075	0.11848991	0.11896539	0.1625566	0.16610051	0.26898029	0.29243741
2.5	0.1	0.13257331	0.13360718	0.18818762	0.19467179	0.31466895	0.35078353
5	0.025	0.00063344507	0.00063362398	0.00076614309	0.00076872083	0.0011513153	0.0011842994
5	0.05	0.00072124025	0.00072240276	0.00094508524	0.00095613626	0.0015301711	0.001623464
5	0.075	0.00081397228	0.00081744639	0.0011226055	0.0011484992	0.0018715629	0.002042953
5	0.1	0.00091232422	0.00091987825	0.0013024543	0.001349831	0.0021935621	0.0024574343
7.5	0.025	4.2685664×10^{-6}	4.2697725×10^{-6}	5.1628804×10^{-6}	5.1802585×10^{-6}	7.7588349×10^{-6}	7.9811999×10^{-6}
7.5	0.05	4.8602529×10^{-6}	4.8680901×10^{-6}	6.3688805×10^{-6}	6.443382×10^{-6}	1.031227×10^{-5}	1.0941213×10^{-5}
7.5	0.075	5.48522×10^{-6}	5.5086411×10^{-6}	7.5653167×10^{-6}	7.7398815×10^{-6}	1.261323×10^{-5}	1.3768672×10^{-5}
7.5	0.1	6.1480702×10^{-6}	6.1989964×10^{-6}	8.7774594×10^{-6}	9.0968543×10^{-6}	1.4783499×10^{-5}	1.6562419×10^{-5}
10	0.025	2.8761395×10^{-8}	2.8769521×10^{-8}	3.4787244×10^{-8}	3.4904337×10^{-8}	5.2278679×10^{-8}	5.3776968×10^{-8}
10	0.05	3.2748152×10^{-8}	3.2800959×10^{-8}	4.2913222×10^{-8}	4.3415212×10^{-8}	6.9483624×10^{-8}	7.3721424×10^{-8}
10	0.075	3.6959154×10^{-8}	3.711696×10^{-8}	5.097476×10^{-8}	5.2150973×10^{-8}	8.4987401×10^{-8}	9.2772737×10^{-8}
10	0.1	4.1425411×10^{-8}	4.176855×10^{-8}	5.9142128×10^{-8}	6.1294202×10^{-8}	9.9610589×10^{-8}	1.115969×10^{-7}

Table 4: Comparison of approximate values of w(x, t) at different fractional order

x	t	ADM α=0.9	MADM α=0.9	ADM α=0.7	MADM α=0.7	ADM α=0.5	MADM α=0.5
2.5	0.025	0.351296880640	0.351312964200	0.385375662930	0.385607404800	0.471840095610	0.474805396300
2.5	0.05	0.374638594810	0.374743105900	0.428133744300	0.429127243400	0.550278925810	0.558666062800
2.5	0.075	0.398123311140	0.398435637600	0.468146449360	0.470474321500	0.617652024230	0.633060178700
2.5	0.10	0.422058898120	0.422738013200	0.507046107030	0.511305330900	0.679297078400	0.703019484100
5	0.025	0.029060559950	0.029060569590	0.031931092470	0.031931231420	0.039260399380	0.039262177330
5	0.05	0.031023736250	0.031023798920	0.035544565080	0.035545160770	0.045937454130	0.045942482950
5	0.075	0.033003132160	0.033003319430	0.038935621180	0.038937016940	0.051687777610	0.051697016150
5	0.1	0.035024188620	0.035024595810	0.042239272280	0.042241826060	0.056958389690	0.056972613350
7.5	0.025	0.002385561080	0.002385561090	0.002621229770	0.002621229850	0.003222986150	0.003222987130
7.5	0.05	0.002546734840	0.002546734880	0.002917899850	0.002917900180	0.003771206190	0.003771208970
7.5	0.075	0.002709242520	0.002709242620	0.003196314560	0.003196315340	0.004243345200	0.004243350310
7.5	0.1	0.002875172530	0.002875172750	0.003467556890	0.003467558300	0.004676101770	0.004676109640
10	0.025	0.000195818850	0.000195818850	0.000215163730	0.000215163730	0.000264558980	0.000264558980
10	0.05	0.000209048810	0.000209048810	0.000239515930	0.000239515930	0.000309559690	0.000309559690
10	0.075	0.000222388270	0.000222388270	0.000262369630	0.000262369630	0.000348315290	0.000348315290
10	0.10	0.000236008650	0.000236008650	0.000284634590	0.000284634590	0.000383838180	0.000383838180

7. Conclusion

In this study, we extended the STADM to the STMADM along with the NIM to develop a series solution for the time fractional Drinfeld-Sokolov-Wilson (DSW) equations. To validate our results, the absolute error is calculated for the series solutions when $\alpha = 1$, where α represents the fractional time derivative and falls within the range $0 < \alpha < 1$, which were obtained with the aid of the STADM and the STMADM. In comparison to the standard STADM, the STMADM typically yields a better approximation with a smaller inaccuracy, this may be because STMADM enhances the convergence rate of the series solution by modifying the decomposition process, allowing the approximate solution to reach the true solution more quickly. This improved convergence requires fewer terms for accuracy, reducing the overall error. Also, present versions of STMADM include higher-order approximations for nonlinear terms, allowing it to represent complex nonlinear behaviors more accurately. Lastly, it has been observed that the solutions produced by the New Iteration Method (NIM) are fairly close to those found by the Shehu Transform Modified Adomian Decomposition Method (STMADM). As a result, the results generated by the Shehu Transform New Iteration Method (STNIM) and STMADM are highly similar. Thus, when applied to time-fractional differential equations, STMADM and STNIM both yield comparable and reliable results.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The authors would like to thank the editor and anonymous reviewers for their comments that helped improve the quality of this work.

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Singh A, Pippal S, Kalia D. Solving the time-fractional DSW equation semi-analytically by combining the Shehu transform with the Adomian decomposition method (ADM), the modified Adomian decomposition method (MADM), and the new iteration method (NIM). Int J Appl Math Numer Res. 2025;1(6):1-17.